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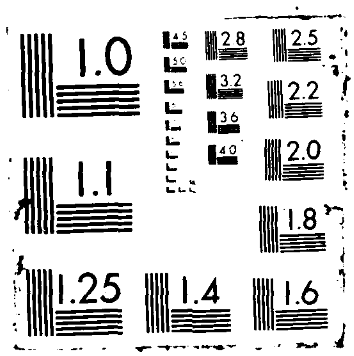
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Angular dispersion of material lines in isotropic turbulence

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The existing body of literature on particle transport by turbulent flow has concentrated on the behavior of spherical particles. However, the dispersed phase in several suspensions of industrial interest consists of nonspherical particles, in particular, flexible slender bodies or threads. The present work considers the problem of disorientation of initially aligned material lines in isotropic turbulence, an idealization which serves to test a general model for particle orientation effects in the transport of threads in turbulent flow. Results obtained from the model agree well with data.

I. INTRODUCTION

The problems of orientation, deformation, and motion of flexible and slender particles in turbulent flow arise in areas such as drag reduction[1,2] by threads of concentrated polymer solutions or by suspended fibers and in various manufacturing operations involving the transport of threads by turbulent airflow. In the present article, we do not attempt to solve this problem in the sense of obtaining a mathematical solution in closed form under particular circumstances; rather, we wish to obtain a closed system of equations which can be used to compute various statistical quantities in arbitrary flow situations. This requires modeling of various terms. If carefully constructed, such models can be more general than solutions resulting from particular assumptions regarding the turbulence, such as that the strain field is white, or Gaussian, or two dimensional, or weak, all of which would permit solutions of the classical form, but none of which is particularly realistic when applied to turbulence. The type of modeling that we propose avoids making assumptions of this type.

In this article, we do not develop a general model; rather, we seek a situation that can be used to calibrate an equation for the evolution of particle orientation, which is one part of a general two-fluid model for the transport of threadlike suspended particles. We present here the part dealing with particle orientation, specialized for a particular set of circumstances. The specialization corresponds to the limit of negligible inertia, perfect flexibility, and perfect extensibility of the particles. In this limit, the problem of particle orientation in turbulent flow reduces to a study of the orientation of material lines in turbulent flow. The effect of turbulence on material lines and surfaces has been theoretically[3,4] and experimentally[5,6] considered in previous investigations. In particular, the angular dispersion of fluid lines has been experimentally studied[6] by introducing lines of hydrogen bubbles in the nearly isotropic turbulence behind a grid. We propose a simple model for the dispersion which is shown to be consistent with the experimental results.

II. MODEL

Let us denote a differential element of the fluid line by **ds** which has components dx_i. The orientation of **ds** is given by the unit vector λ whose components are

$$\lambda_i = \frac{dx_i}{\sqrt{dx_j\, dx_j}}. \tag{1}$$

We need to derive an equation for the evolution of λ_i with time. Such an equation can be obtained by taking the substantial derivative of Eq. (1). After differentiating Eq. (1), expressing the time derivatives of dx_i and $dx_j\, dx_j$ in terms of the spatial derivatives of the fluid velocity u_i, and simplifying, we obtain

$$\frac{D\lambda_i}{Dt} = \lambda_j u_{i,j} - \lambda_i \lambda_j \lambda_k u_{j,k}, \tag{2}$$

which if rewritten as

$$\lambda_j u_{i,j} = \frac{D\lambda_i}{Dt} + \lambda_i (\lambda_j \lambda_k u_{j,k}) \tag{3}$$

can be seen to be an expression for the relative motion between adjacent points on **ds** in terms of the rotation of **ds** and a stretching along **ds**. It should be noted that when the two-fluid model is used to describe particle transport in a carrier fluid, Eq. (2) is the equation for the evolution of particle orientation.

Equation (2) is somewhat complicated by the requirement that λ_i be a unit vector, which is responsible for the last term in the equation. We could, of course, work with the unnormalized vector, which would make modeling somewhat easier. In the full problem, however, the normalized vector λ_i appears very often in the momentum equations. Using the unnormalized vector would simplify Eq. (2) and greatly complicate all the other equations. We feel that the use of the normalized vector produces the greatest overall simplicity.

Now λ_i and u_i are decomposed into their means Λ_i and and U_i and their fluctuations λ_i' and u_i' as follows:

$$\lambda_i = \Lambda_i + \lambda_i', \tag{4}$$

$$u_i = U_i + u_i'. \tag{5}$$

On squaring and averaging Eq. (4), we obtain

$$1 = \Lambda_i \Lambda_i + \overline{\lambda_i' \lambda_i'}. \tag{6}$$

Thus, although λ_i is a unit vector, Λ_i is not a unit vector.

In the case of the problem when both the turbulence and the suspended matter are homogeneous and there is no mean shear, the averaged form of Eq. (2) reduces to

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$$\begin{aligned}\partial_t(\Lambda_i) = {} & \big[\, \overline{\lambda'_j u'_{i,j}} - \overline{u'_j \lambda'_{i,j}} - \Lambda_i\Lambda_j\, \overline{\lambda'_k u'_{j,k}} \\ & - \Lambda_i\Lambda_k\, \overline{\lambda'_j u'_{j,k}} - \Lambda_j\Lambda_k\, \overline{\lambda'_i u'_{j,k}}\,\big] \\ & - (\, \overline{\lambda'_i\lambda'_j\lambda'_k u'_{j,k}}\,) - \{\Lambda_i\, \overline{\lambda'_j\lambda'_k u'_{j,k}} \\ & + \Lambda_j\, \overline{\lambda'_i\lambda'_k u'_{j,k}} + \Lambda_k\, \overline{\lambda'_i\lambda'_j u'_{j,k}}\,\}. \qquad (7)\end{aligned}$$

The third-order correlations in curly braces in Eq. (7) may be dropped if we assume that the fluid line statistics are joint normal. We assume the carrier fluid turbulence to be isotropic; hence we expect the remaining correlations in Eq. (7) to be axisymmetric around Λ_i. We also assume that the fluid lines are initially aligned in a particular direction; hence all the correlations must be 0 at $t = 0$. Eventually the turbulence completely destroys the orientation of the fluid lines and therefore the correlations must also tend to 0 as $t \to \infty$. From Eq. (6) it can be seen that the magnitude of λ'_i is $(1-\Lambda^2)^{1/2}$, where Λ^2 denotes $\Lambda_i\Lambda_i$. We normalize by the appropriate power of the magnitude so that the correlations will behave correctly when $\Lambda = 1$, which occurs at $t = 0$, and when $\Lambda \to 0$, which occurs as $t \to \infty$. Furthermore, in Eq. (7) we assume that there is a time scale associated with the second-order correlations in square brackets and another time scale corresponding to the third-order correlation in parentheses.

The above considerations lead to the following models for the terms on the right-hand side of Eq. (7):

$$[\cdot] = -\Lambda_i(1-\Lambda^2)^{1/2} f, \qquad (8)$$

where $\Lambda^2 = \Lambda_j\Lambda_j$ and $f =$ inverse time scale of $[\cdot]$;

$$(\cdot) = -\Lambda_i(1-\Lambda^2)^{3/2} n f \qquad (9)$$

where n is the ratio of the inverse time scales of $(\cdot)$ and $[\cdot]$;

$$\{\cdot\} = 0. \qquad (10)$$

After substituting Eqs. (8)–(10) into Eq. (7) and forming an evolution equation for Λ^2, we obtain

$$\partial_t(\Lambda^2) = -2\Lambda^2(1-\Lambda^2)^{1/2} f[1 - n(1-\Lambda^2)]. \quad (11)$$

In Eq. (11), since $\partial_t(\Lambda^2) = 0$ at $t = 0$, we require $\partial_t^2(\Lambda^2) < 0$ at $t = 0$ in order that Λ^2 decay with time. This constraint is assured by a value of $\frac{1}{2}$ for the fractional exponent in Eq. (11); no other value would satisfy this constraint.

Since in our simple problem we expect f to scale with the inverse time scale of the fluid turbulence, we take

$$f = \alpha\epsilon/q^2, \qquad (12)$$

where ϵ is the isotropic dissipation rate and q^2 is twice the turbulent kinetic energy. The expression for the inertial frequency scale of the decaying turbulence behind the grid in the modeled experiment[6] is

$$\epsilon/q^2 = 0.625/(2+t), \qquad (13)$$

where t is the time elapsed in seconds after the introduction of the material lines into the flow.

Substituting Eqs. (12) and (13) into Eq. (11), we obtain

$$\partial_t(\Lambda^2) = \frac{-1.25\alpha\Lambda^2(1-\Lambda^2)^{1/2}[1-n(1-\Lambda^2)]}{2+t}. \qquad (14)$$

Equation (14), along with the adjustable parameters α and n, is an equation that describes the progressive disorientation of the fluid lines with time. The model parameters α and n are chosen so that the behavior of $\Lambda^2(t)$ obtained by solving Eq. (14) matches with the experimental data.[6] The initial condition for Eq. (14) is

$$\Lambda^2(t=0) = 1. \qquad (15)$$

III. RESULTS

In the experimental setup,[6] the flow was in the one-direction, with the lines introduced initially in the two-direction, as shown in Fig. 1. The results of that experiment were presented as a plot of $\overline{(\theta'_3)^2}(t^*)$, where $\overline{(\theta'_3)^2}$ is the variance of θ_3 (see Fig. 1). The independent variable in the experimental plot was t^*, which represents time normalized in a manner which accounts for the decay of the grid turbulence and is given by

$$t^* = 181[0.707 - (16+8t)^{-0.125}]. \qquad (16)$$

Equation (14), along with its initial condition (15), was solved analytically for $\Lambda^2(t)$. In order to compare the model with experimental data we need an expression for $\overline{(\theta'_3)^2}$ in terms of Λ^2. We can write

$$\theta'_3 = -\arcsin\left(\lambda'_1/\sqrt{1-\lambda'^2_3}\right), \qquad (17)$$

where λ'_1 and λ'_3 are the fluctuations in the components of the fluid orientation vector along the one and three axes, respectively.

After squaring Eq. (17); expanding its right-hand side in a Taylor series up to fourth order in λ'_1 and λ'_3, assuming that the distributions of λ'_1 and λ'_3 are statistically independent; and averaging, we obtain

$$\overline{(\theta'_3)^2} \simeq \overline{\lambda'^2_1} + \overline{\lambda'^2_1}\;\overline{\lambda'^2_3} + \overline{\lambda'^4_1}/3. \qquad (18)$$

In order to express $\overline{(\theta'_3)^2}$ in terms of Λ^2 we now need expressions for the correlations on the right-hand side of Eq. (18) in terms of the mean orientation vector Λ_i. An equation for $\overline{\lambda'_i\lambda'_j}$ can be derived as follows. In the homogenous problem which is considered in the present work, the anisotropy in $\overline{\lambda'_i\lambda'_j}$ is due to the anisotropy in $\Lambda_i\Lambda_j$; hence

$$\overline{\lambda'_i\lambda'_j} = (1-\Lambda^2)[\delta_{ij}/3 + \beta(\Lambda_i\Lambda_j - \Lambda^2\delta_{ij}/3)]. \qquad (19)$$

To compute β, consider almost aligned fibers; $\Lambda_i = (1-\epsilon, 0, 0)$, where ϵ is a small parameter. In such a case

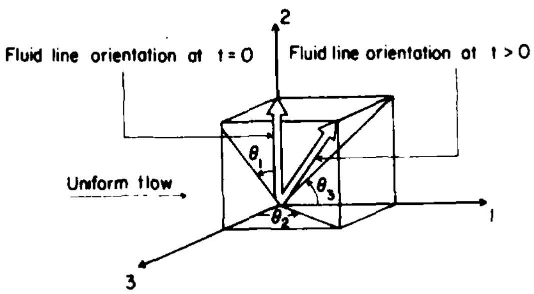

FIG. 1. Notation for the fluid line orientation.

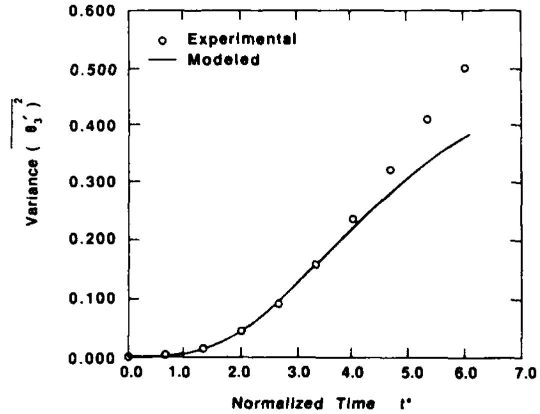

FIG. 2. The transient behavior of angular dispersion.

Eq. (19) gives a diagonal form for $\overline{\lambda_i' \lambda_j'}$, which to first order in ϵ is

$$\overline{\lambda_1'^2} = 2\epsilon(\tfrac{1}{3} + \tfrac{2}{3}\beta), \quad \overline{\lambda_2'^2} = 2\epsilon(\tfrac{1}{3} - \tfrac{1}{3}\beta), \quad \overline{\lambda_3'^2} = 2\epsilon(\tfrac{1}{3} - \tfrac{1}{3}\beta). \tag{20}$$

We require that $\beta \leqslant 1$ in order that $\overline{\lambda_2'^2}$ and $\overline{\lambda_3'^2}$ be always non-negative. It seems plausible that β is a weak function of Λ^2; hence we assume that β is a constant and set it equal to 1. Taking $\beta = 1$ in Eq. (19) results in

$$\overline{\lambda_i' \lambda_j'} = (1 - \Lambda^2)[\delta_{ij}/3 + (\Lambda_i \Lambda_j - \Lambda^2 \delta_{ij}/3)]. \tag{21}$$

In the present case, due to symmetry, $\Lambda_1 = 0$ and $\Lambda_3 = 0$ for all time. Therefore, from Eq. (21),

$$\overline{\lambda_1'^2} = \overline{\lambda_3'^2} = (1 - \Lambda^2)^2/3. \tag{22}$$

If we assume that λ_1' has a Gaussian distribution, then

$$\overline{\lambda_1'^4} = 3(\overline{\lambda_1'^2})^2. \tag{23}$$

Substituting Eqs. (22) and (23) into Eq. (18), we obtain

$$\overline{(\theta_3')^2} \simeq (1 - \Lambda^2)^2/3 + 2(1 - \Lambda^2)^4/9. \tag{24}$$

Numerical results of $\overline{(\theta_3')^2}(t^*)$ were obtained for various values of the parameters α and n. The choice of

$$\alpha = 9.4 \quad \text{and} \quad n = 0.5 \tag{25}$$

gives a good fit between the prediction of the model and the experimental data, as shown in Fig. 2. It should be noted that Eq. (24) is obtained by a Taylor expansion of Eq. (17) for small λ_i' and therefore we do not expect a good match for large time. There is no reason, however, to expect Eq. (11) to be any less accurate for large time. Substituting Eqs. (12) and (25) into Eq. (11), we obtain

$$\partial_t(\Lambda^2) = -18.8\epsilon\Lambda^2(1 - \Lambda^2)^{1/2}[1 - 0.5(1 - \Lambda^2)]/q^2. \tag{26}$$

In conclusion, we have been able to derive an ordinary differential equation, Eq. (26), for the mean orientation vector Λ_i, which characterizes the disorienting effect of turbulence on initially aligned material lines in an isotropic turbulent fluid. Perhaps more valuable, we have used this special case to calibrate a model which may have more general utility.

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[1] J. L. Lumley and I. Kubo, in *IUTAM Symposium, Essen 1984: Influence of Polymer Additives on Velocity and Temperature Fields*, edited by B. Gampert (Springer, Berlin, 1985), p. 3.
[2] J. L. Lumley, Phys. Fluids **20**, S64 (1977).
[3] G. K. Batchelor, Proc. R. Soc. London Ser. A, **213**, 349 (1952).
[4] W. H. Reid, Proc. Cambridge Philos. Soc. **51**, 350 (1955).
[5] S. Corrsin and M. J. Karweit, J. Fluid Mech. **39**, 87 (1969).
[6] S. Corrsin and M. J. Karweit, J. Fluid Mech. **55**, 289 (1972).

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